

Highly super-Chandrasekhar white dwarfs in an extensive GRMHD framework

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Our consistent effort to unravel the mystery of super-Chandrasekhar white dwarfs (WDs), by exploiting the potential of magnetic fields, has brought this topic considerable attention. This is also evident from the recent surge in the corresponding literature. In the present work, by means of full-scale general relativistic magnetohydrodynamic (GRMHD) numerical analysis, we confirm the existence of stable, highly magnetized, significantly super-Chandrasekhar WDs having mass exceeding 3 solar mass. We have explored various possible field configurations, namely, poloidal, toroidal and mixed, by self-consistently incorporating the departure from spherical symmetry induced by a strong magnetic field. Such super-Chandrasekhar WDs can be ideal progenitors of peculiar, over-luminous type Ia supernovae.

Keywords: stars: magnetic fields; white dwarfs; stars: massive; gravitation; MHD; supernovae: general

1. Introduction

With the aim of obtaining a fundamental basis behind the formation of super-Chandrasekhar white dwarfs (WDs), Mukhopadhyay and his collaborators^{1–6} initiated the exploration of highly magnetized WDs and their new mass-limit, significantly exceeding the Chandrasekhar limit of $1.44M_{\odot}$.⁷ These WDs are ideally suited to be the progenitors of peculiar, overluminous, type Ia supernovae, e.g. SN 2003fg, SN 2006gz, SN 2007if, SN 2009dc,^{8,9} which are best explained by invoking the explosion of super-Chandrasekhar WDs having mass $2.1 - 2.8M_{\odot}$. Along with the fact that several WDs have been discovered with surface fields $10^5 - 10^9$ G, it has also been known that magnetized WDs tend to be more massive than their non-magnetized counterparts.¹⁰ Such observations motivate the theoretical investigation of the effect of a strong interior magnetic field on the mass of a WD.

In this context, we mention that our previous attempts at obtaining highly magnetized super-Chandrasekhar WDs, assumed *a priori* spherical symmetry. While that may indeed be the case for certain magnetic field geometries, in general, highly magnetized WDs tend to be deformed due to magnetic tension. With each new step we have scientifically progressed towards a more rigorous model — starting from a simplistic Newtonian, spherically symmetric, constant field model and culminating in a model with self-consistent departure from spherical symmetry by general

relativistic magnetohydrodynamic (GRMHD) formulation, which we explain in the present work (also, see Ref. 11). We appropriately modify the *XNS* code,^{12,13} which has so far been used only to model strongly magnetized neutron stars, to compute equilibrium configurations of static, strongly magnetized WDs in the GR framework, for the first time in the literature to the best of our knowledge.

2. Numerical set-up

For a detailed description of the underlying GRMHD equations, the magnetic field geometries, the numerical technique employed by the *XNS* code and the values of various code parameters, we refer the readers to Refs. 11-13.

We construct axisymmetric WDs in spherical polar coordinates (r, θ, ϕ) , to self-consistently account for the deviation from spherical symmetry due to a strong magnetic field, which generates an anisotropy in the magnetic pressure.¹⁴ A uniform computational grid is used along both the radial r and polar θ co-ordinates, the number of grid points being typically $N_r = 500$ and $N_\theta = 100$ respectively. Even higher resolution runs (for e.g. with $N_r = 1000$ and $N_\theta = 500$) require more computational time but do not lead to any significant change in the results.

In this work, we focus on the equilibrium solutions of high density, magnetized, relativistic WDs, which can be described by a polytropic equation of state (EoS) $P = K\rho^\Gamma$, where P is the pressure and ρ the density, such that the adiabatic index $\Gamma \approx 4/3$ and the constant K is same as that obtained by Chandrasekhar.⁷ Hence, we neglect the possible effect of Landau quantization on the above EoS which could arise due to a strong magnetic field $B > B_c$, where $B_c = 4.414 \times 10^{13}$ G, is the critical magnetic field.¹ We recall that the maximum number of Landau levels ν_m occupied by electrons in the presence of a magnetic field is given by equation (10) of Ref. 1. The range of central density and maximum magnetic field strength inside the WDs considered in this work are $10^{10} \lesssim \rho_c \lesssim 10^{11}$ gm/cm³ and $10^{13} \lesssim B_{\max} \lesssim 10^{15}$ G respectively. Consequently, $\nu_m \gtrsim 20$ for this range of ρ_c and B_{\max} , which is large enough not to significantly modify the value of Γ we choose, hence justifying our assumption.

3. Results with different magnetic field configurations

We now explore the effect of various magnetic field geometries on the structure and properties of WDs. For a fiducial model, we choose a non-magnetized WD with $\rho_c = 2 \times 10^{10}$ gm/cm³. It has a baryonic mass $M_0 = 1.416 M_\odot$ and equatorial radius $R_{eq} = 1221.94$ km, and is perfectly spherical with $R_p/R_{eq} = 1$, R_p being the polar radius (note that for the definitions of all global physical quantities characterizing the solutions in this work, we refer to Appendix B of Ref. 13).

Figures 1(a) and (c) portray the distribution of baryonic density and magnetic field strength respectively, for the fiducial WD having a purely toroidal magnetic field configuration $\vec{B} = B_\phi \hat{\phi}$. The maximum magnetic field strength inside this

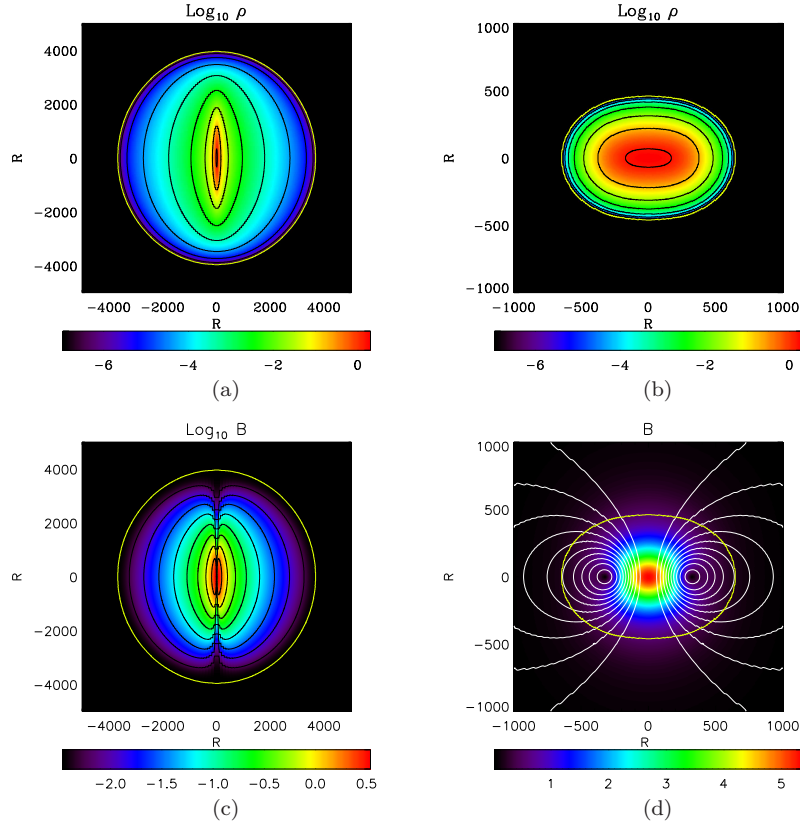


Fig. 1. Purely toroidal configuration: iso-contours of (a) baryonic density and (c) magnetic field strength. Purely poloidal configuration: iso-contours of (b) baryonic density and (d) magnetic field strength, superimposed with dipolar magnetic field lines (in white). R is in units of $GM_{\odot}/c^2 = 1.476$ km, ρ in units of 10^{10} gm/cm³ and B in units of 10^{14} G. The yellow curve in each panel represents the stellar surface.

WD is $B_{\text{max}} = \sqrt{B_{\phi} B^{\phi}} = 3.41 \times 10^{14}$ G. Interestingly, this is a highly super-Chandrasekhar WD, having $M_0 = 3.413M_{\odot}$. Note that the value of the surface magnetic field does not affect this result, as long as it is $\lesssim 10^{11}$ G, which is satisfied in this case. In this context, we mention that the detection of very high surface magnetic fields $\gtrsim 10^9$ G is very difficult due to the featureless spectrum.¹⁰ Very importantly, the ratio of the total magnetic energy to the total gravitational binding energy, $E_{\text{mag}}/E_{\text{grav}} = 0.3045$ (which is much < 1). WDs with even smaller $E_{\text{mag}}/E_{\text{grav}}$ are also found to be highly super-Chandrasekhar (see Figs. 3a and d). This argues for the WDs to be stable (see, e.g. Ref. 15). The radii ratio, $R_p/R_{\text{eq}} = 1.074$ (which is slightly > 1), indicates a net prolate deformation in the shape caused due to a toroidal field geometry. Figure 1(a) shows that although the central iso-density contours are compressed into a highly prolate structure, the

outer layers expand, giving rise to an overall quasi-spherical shape. Interestingly, this justifies the earlier spherically symmetric assumption in computing models of at least certain strongly magnetized WDs.^{1,2,4}

Figures 1(b) and (d) show the distribution of baryonic density and magnetic field strength superimposed by magnetic field lines respectively, for the fiducial WD having a purely poloidal magnetic field configuration $\vec{B} = B_r \hat{r} + B_\theta \hat{\theta}$. The maximum magnetic field strength attained at its center is $B_{\max} = \sqrt{B_r B^r + B_\theta B^\theta} = 5.34 \times 10^{14}$ G, which also leads to a significantly super-Chandrasekhar WD having $M_0 = 1.771 M_\odot$. The WD is highly deformed with an overall oblate shape and $R_p/R_{eq} = 0.7065$, which is expected to be stable because its $E_{mag}/E_{grav} = 0.1138$, which is very much < 1 .¹⁵

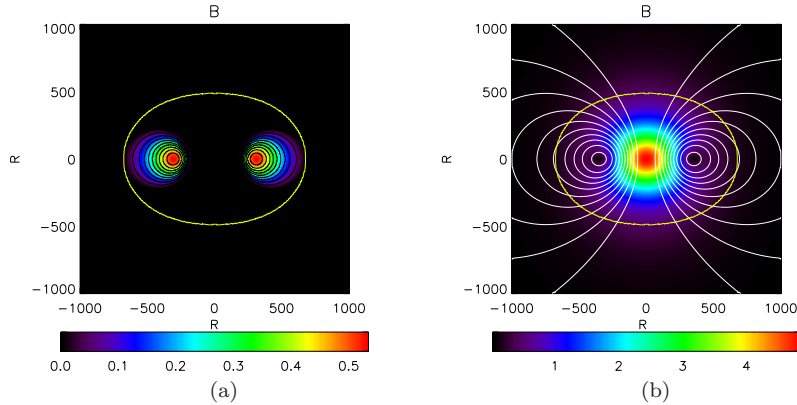


Fig. 2. Mixed field or twisted torus configuration: magnetic field strength of (a) toroidal component and (b) poloidal component.

Purely toroidal and poloidal field configurations are believed to be subjected to MHD instabilities,¹⁶ which possibly rearranges the field into a mixed configuration.¹⁷ Hence, for completeness, we also construct equilibrium models of WDs with a mixed magnetic field configuration. In Figure 2, we present the results for the fiducial WD having the so-called *twisted torus configuration*, which compares the distribution of the toroidal and poloidal components of the magnetic field. This again results in a significantly super-Chandrasekhar WD having $M_0 = 1.754 M_\odot$. The poloidal component attains $B_{\max} = 4.82 \times 10^{14}$ G at the center, while the ring-like toroidal component is an order of magnitude smaller. The WD assumes a highly oblate shape with $R_p/R_{eq} = 0.719$, resembling the purely poloidal case in all its attributes, and is again expected to be stable having $E_{mag}/E_{grav} = 0.1126 < 1$. In a toroidal dominated mixed field configuration,¹⁸ more massive super-Chandrasekhar WDs could be possible and is worth further exploration. In this context, we mention that the results in this work have also been reproduced by Bera & Bhattacharya,¹⁹

albeit without any novel contribution to the topic.

We also construct equilibrium sequences of magnetized WDs pertaining to different field geometries for the fiducial case with $\rho_c = 2 \times 10^{10} \text{ gm/cm}^3$. Figure 3 shows the variations of different physical quantities as functions of B_{max} . The two most important revelations of Figure 3 are — (1) the WD mass increases with an increase in magnetic field for all the three field configurations discussed above (see Figure 3a), eventually leading to highly super-Chandrasekhar WDs and (2) the magnetic energy remains (significantly) sub-dominant compared to the gravitational binding energy for all the cases, since $E_{\text{mag}}/E_{\text{grav}} < 1$ always (see Figure 3d), which argues for the possible existence of stable, highly magnetized super-Chandrasekhar WDs.

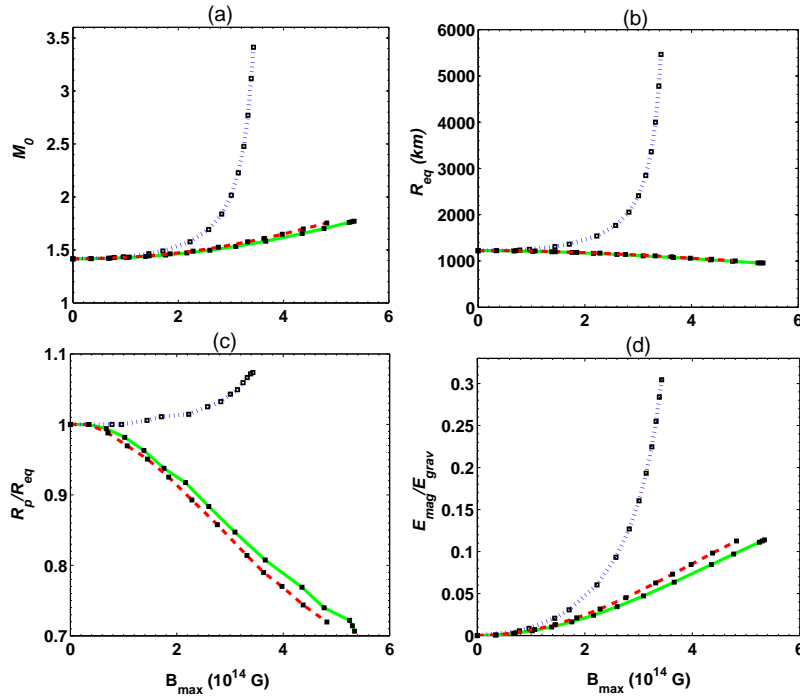


Fig. 3. Equilibrium sequences of magnetized WDs with fixed $\rho_c = 2 \times 10^{10} \text{ gm/cm}^3$. (a) M_0 , (b) R_{eq} , (c) R_p/R_{eq} and (d) $E_{\text{mag}}/E_{\text{grav}}$, as functions of B_{max} . The solid (green), dotted (blue) and dashed (red) curves represent, respectively, WDs having purely poloidal, purely toroidal, and twisted torus field configurations. M_0 is in units of M_{\odot} . The filled boxes represent individual WDs.

4. Conclusions

Since our foray into this topic, we have been persistent with our message that the versatile nature of magnetic field is paramount in the revelation of significantly

super-Chandrasekhar WDs, irrespective of its nature of origin: quantum, classical and/or general relativistic — which is re-emphasized in the present work.

By carrying out extensive, self-consistent, GRMHD numerical analysis of magnetized WDs, we have reestablished the existence of highly super-Chandrasekhar, stable WDs. In order to self-consistently study the anisotropic effect of a strong magnetic field, we have explored various geometrical field configurations, namely, purely toroidal, purely poloidal and twisted torus configurations. Interestingly, we have obtained significantly super-Chandrasekhar magnetized WDs for all the cases, having mass $1.7 - 3.4M_{\odot}$, and that too at relatively lower magnetic field strengths when the deviation from spherical symmetry is considered — as already speculated in our earlier work.^{1,6} These WDs can be ideal progenitors of the aforementioned peculiar, overluminous type Ia supernovae. Our work also establishes the necessity of a general relativistic formalism over a Newtonian approach while constructing models of magnetized super-Chandrasekhar WDs.

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